

Assessing symmetry of financial returns series

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Abstract

Testing symmetry of a probability distribution is a common question arising from applications in several fields. Particularly, in the study of observables used in the analysis of stock market index variations, the question of symmetry has not been fully investigated by means of statistical procedures. In this work a distribution-free test statistic T_n for testing symmetry, derived by Einmahl and McKeague, based on the empirical likelihood approach, is used to address the study of symmetry of financial returns. The asymptotic points of the test statistic T_n are also calculated and a procedure for assessing symmetry for the analysis of the returns of stock market indices is presented.

Key words: Econophysics, Statistical Test, Symmetry Test, Returns Distribution, Gain/Loss Asymmetry

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1 Introduction

The gain/loss asymmetry of stock price variations is considered as one of the stylized facts of financial time series [1] and its nature is of great and current interest [2]. In particular, and even though it has been researched for many years, the study of the symmetry of the unconditional distribution of financial returns remains as an important subject. For instance, in reference [3] conditions under which the distribution of ensemble returns becomes asymmetric

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are reported. On the other hand, [4] has analyzed returns of a big sample of diverse financial indices without finding important symmetry deviations.

Then, due to the importance of this subject, the assumption of symmetry of the distribution of returns should be supported by means of objective distribution-free statistical procedures.

In next section of this paper, we present and review a distribution-free test statistic T_n for testing symmetry, derived by Einmahl and McKeague [5], based on the empirical likelihood approach. In section 3 we show our numerical calculation of the asymptotic distribution of the T_n statistic derived by simulation in [5]. In section 4 we present a procedure for assessing symmetry of returns distribution by using the statistic T_n and illustrating it with data of the Mexican Stock Market Index IPC (*Índice de Precios y Cotizaciones* or by its English meaning Prices and Quotations Index) and the Dow Jones Industrial Average Index DJIA.

2 The T_n Statistic

An approach to omnibus hypothesis testing based on the empirical likelihood method has been published in a very interesting paper by Einmahl and McKeague [5]. For testing the null hypothesis of symmetry about zero, $H_0 : F(0 - x) = 1 - F(x - 0)$, for all $x > 0$ based on a sample X_1, \dots, X_n of independent and identically distributed random variables with common absolutely continuous distribution function F , they derived as a test statistic, the quantity:

$$T_n = -2 \int_0^\infty \log H(x) dG_n(x) = -\frac{2}{n} \sum_{i=1}^n \log H(|X_i|). \quad (1)$$

G_n denotes here the empirical distribution function of the $|X_i|$ and:

$$\begin{aligned} \log H(x) = & nF_n(-x) \log \frac{F_n(-x) + 1 - F_n(x-)}{2F_n(-x)} \\ & + n[1 - F_n(x-)] \log \frac{F_n(-x) + 1 - F_n(x-)}{2[1 - F_n(x-)]}, \end{aligned}$$

where notation means $F_n(-x) := F_n(0 - x)$ and $F_n(x-) := F_n(x - 0)$.

The limiting distribution was found by proving that T_n converges weakly to:

$$T_n \xrightarrow{D} \int_0^1 \frac{W(t)^2}{t} dt, \quad (2)$$

where W denotes a standard Wiener process.

3 Calculation of the Asymptotic Distribution of T_n

The asymptotic percentage points of the limiting distribution of T_n were obtained here using (see for example [6]) the series representation:

$$T_n \xrightarrow{D} \sum_{i=1}^{\infty} \lambda_i \nu_i,$$

where ν_1, ν_2, \dots are independent chi-squared random variables, with one degree of freedom, and $\lambda_1, \lambda_2, \dots$ are the eigenvalues of the integral equation:

$$\int_0^1 \sigma(s, t) f_i ds = \lambda_i f_i(t), \quad (3)$$

with $\sigma(s, t)$ denoting the covariance function of the process $\frac{W(t)}{\sqrt{t}}$.

Due to the difficulty of solving analytically equation (3), the asymptotic percentage points of the distribution of T_n were found numerically; using $k = 100$ equally spaced points in the interval $(0, 1)$ the integral was approximated in order to solve equation (3). Similarly, a k by k grid on $(0, 1) \times (0, 1)$ was constructed to evaluate the covariance function $\sigma(s, t)$ and the eigenvalue problem solved to estimate $\lambda_1, \dots, \lambda_k$. Using these approximations, the asymptotic percentage points were calculated using Imhof's method [7]. The above procedure was repeated for $k=200$ and $k=300$, and the results compared. As it can be seen from table 1, the percentage points obtained are almost identical except for a few discrepancies not greater than one unit in the third decimal figure. These results are consistent with those obtained by simulation and reported in Einmahl and McKeague's paper.

4 Proposed approach and examples

Given a set of observations from an unknown probability distribution, if the symmetry point is known, a statistical procedure (as the one described above) can be used to test the symmetry of the distribution around that point. However, when the symmetry point is unknown, it might happen that the test would lead us to the rejection of this assumption, even when the distribution is symmetric; this would be the case when the symmetry point is incorrectly specified in the test.

Let us denote by $\{S_t\}$ the stock index process and by $R_t = \log S_t - \log S_{t-\Delta t}$ its returns or logarithmic increments during a certain time interval Δt . The "shifted returns" are also defined as $R_t(c) = R_t - c$, where c denotes a real number. Finally, let us denote by $T_n(c)$ the value of the test-statistic T_n calculated from $R_1(c), \dots, R_N(c)$ for a particular value of c .

In the following, we will mean by a *plausible* value of the symmetry point, (for a significance level α) any real number c_0 , such that $T_n(c_0) < T(\alpha)$ where $T(\alpha)$ denotes the α -level upper point of the distribution of T_n .

Cumulative Probability	Percentage point ($k = 300$)	Percentage point ($k = 200$)
0.50	0.659	0.659
0.75	1.258	1.258
0.85	1.768	1.768
0.90	2.200	2.200
0.95	2.983	2.982
0.975	3.798	3.797
0.990	4.909	4.908
0.995	5.768	5.767
0.999	7.803	7.803

Table 1

Asymptotic percentage points of T_n calculated numerically. It can be seen from two columns values that numerical convergence of T_n is very fast.

Using a similar approach to that of constructing confidence regions, a plot of $T_n(c)$ versus c can be used to identify a *plausible* set of values of the unknown symmetry point c in the sense that, for a given significance level α , the interval would contain the set of all possible values of c which would not lead to the rejection of the null hypothesis of symmetry for the probability distribution of the random variable R_t .

In order to illustrate the procedure, we present our analysis for two data sets:

- (1) DJIA Daily closing values from October 30, 1978 to October 20, 2006.
- (2) IPC Daily closing values for the same period.

For each data set, the shifted returns R_t were obtained, and the plots produced using the procedure described above.

In figure 1, it is shown the plot from the Dow Jones index data, including the lines $y = 4.909$, $y = 2.983$ and $y = 2.200$, which correspond to the asymptotic 0.99, 0.95 and 0.90 percentiles of distribution of the T_n statistic, from table 1. As it can be seen, for a significance level $\alpha = 0.10$ (or lower), it is possible to find an interval of plausible values for the unknown point of symmetry which would not lead us to the rejection of the assumption of symmetry. Approximately, for $\alpha = 0.10$, any value within the interval $(2.6 \times 10^{-4}, 6.2 \times 10^{-4})$ can be statistically considered as a point around which the distribution of the returns is symmetric.

Figure 2 shows the symmetry plot for the returns obtained from the Mexican IPC index data. Considering the 90% percentage line, we find that an interval of plausible values for the point of symmetry can be found; approximately the interval $(1.16 \times 10^{-3}, 1.74 \times 10^{-3})$ would be a 90% confidence-interval

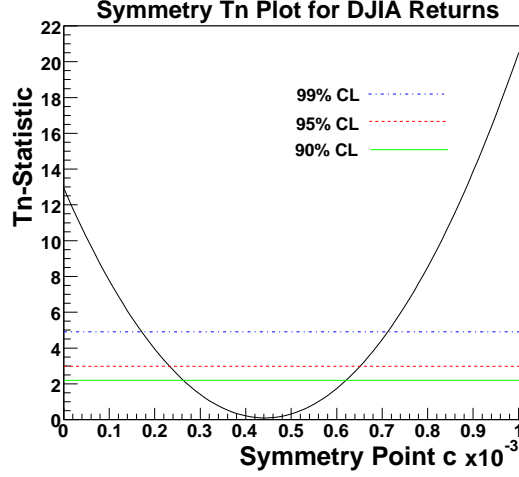


Fig. 1. Plot of statistic $T_n(c)$ versus selected values of the symmetry point c for the Dow Jones return series data. Horizontal straight lines correspond to the 99, 95 and 90 upper percentage points, as indicated

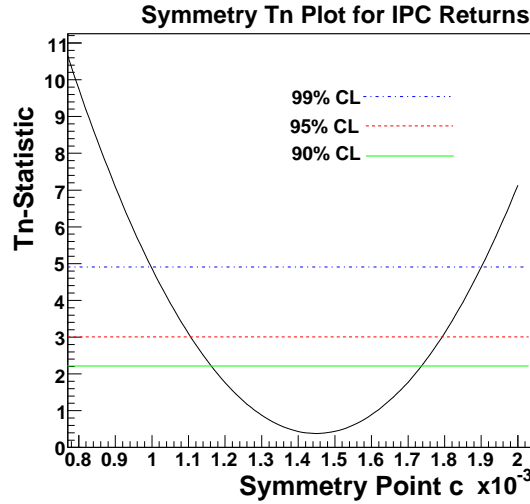


Fig. 2. Plot of statistic $T_n(c)$ versus selected values of the symmetry point c for the IPC return series data. Horizontal straight lines correspond to the 99, 95 and 90 upper percentage points.

for the unknown point of symmetry; that is, if we choose any value for the symmetry point within that interval, the statistic T_n would not lead to the rejection of the hypothesis of symmetry around the choosen point. Again, our assessment would be that, for a given significance level $\alpha = 0.10$ (or lower), there exists a set of plausible values for which the assumption of symmetry can be statistically supported.

It must be remarked that the approach discussed here is not equivalent to that of maximizing a test-statistic as it has been the case, for example, in [8] or [9] and [10]. The reasoning behind our assessment is based on the idea

that whenever there exists a plausible value for the point of symmetry, this assumption can be statistically sustained.

5 Conclusions

A procedure for assessing the assumption of symmetry, for the probability distribution function of returns, has been presented. The approach is based on determining, statistically, whether or not, a set of plausible values for the unknown symmetry point can be found. Two examples were discussed to illustrate the approach, analyzing returns data from the Dow Jones and the Mexican IPC stock market indices. In both cases, sets of plausible values for the point of symmetry could be found, so that the assumption of symmetry can be statistically supported.

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